



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$u = \frac{d_1 d_4 - d_2 d_3 \pm \sqrt{[(d_1 d_4 - d_2 d_3)^2 + 4(d_1 d_3 - d_2^2)(d_2^2 - d_2 d_4)]}}{2(d_1 d_3 - d_2^2)}.$$

From (13) we find v . Then from (10) we find z . From (6), y , and from (1), x .

351. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

$$\begin{aligned} \text{Solve, } y^2 + yz + z^2 &= a^2 \dots (1). \\ z^2 + zx + x^2 &= b^2 \dots (2). \\ x^2 + xy + y^2 &= c^2 \dots (3). \end{aligned}$$

I. Solution by J. A. COLSON, Searsport, Maine.

$$\begin{aligned} b^2 - c^2 &= (z - y)(x + y + z). \quad \therefore (b^2 - c^2)x = (zx - xy)(x + y + z). \\ c^2 - a^2 &= (x - z)(x + y + z). \quad \therefore (c^2 - a^2)y = (xy - yz)(x + y + z). \\ a^2 - b^2 &= (y - x)(x + y + z). \quad \therefore (a^2 - b^2)z = (yz - zx)(x + y + z). \\ \therefore (b^2 - c^2)x + (c^2 - a^2)y + (a^2 - b^2)z &= 0. \end{aligned}$$

For convenience, put $b^2 - c^2 = f$, $c^2 - a^2 = g$, and $a^2 - b^2 = h$. Then $f + g + h = 0$, and $fx + gy + hz = 0$.

$$\therefore z = -\frac{fx + gy}{h}, \text{ and } x + y + z = x + y - \frac{fx + gy}{h} = \frac{(h - f)x + (h - g)y}{h}.$$

$$\therefore a^2 - b^2 = h = (y - x)(x + y + z) = (y - x) \frac{(h - f)x + (h - g)y}{h}.$$

$$\therefore h^2 = (f - h)x^2 + (g - f)xy + (h - g)y^2.$$

But from (3) we have $y^2 = c^2 - x^2 - xy$.

Hence, $h^2 = c^2(h - g) + (f + g - 2h)x^2 + (2g - f - h)xy = c^2(h - g) - 3hx^2 + 3gxy$.

$$\therefore y = \frac{3hx^2 + h^2 + c^2(g - h)}{3gx}.$$

Substitute in (3), and we have

$$x^2 + \frac{3hx^2 + h^2 + c^2(g - h)}{3g} + \frac{[3hx^2 + h^2 + c^2(g - h)]^2}{9g^2x^2} - c^2 = 0.$$

Hence, clearing of fractions and uniting, we have,

$$\begin{aligned} 9(g^2 + gh + h^2)x^4 - 3[c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)]x^2 \\ + [h^2 + c^2(g - h)]^2 = 0. \end{aligned}$$

$$\begin{aligned} \therefore 36(g^2 + gh + h^2)^2x^4 - 12(g^2 + gh + h^2)[c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)]x^2 \\ + [c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)]^2 = [c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)]^2 - 4(g^2 \\ + gh + h^2)[h^2 - c^2(g - h)]^2 = 9c^4g^2h^2 - 6c^2g^2h^2(2g + h) - 3g^2h^4. \end{aligned}$$

$$\begin{aligned} \therefore 6(g^2 + gh + h^2)x^2 - [c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)] \\ = \pm gh\sqrt{9c^4 - 6c^2(2g + h) - 3h^2}. \end{aligned}$$

Giving g and h their original values, we have

$$6(a^4 + b^4 + c^4 - b^2c^2 - c^2a^2 - a^2b^2)x^2 = 2b^6 + 2c^6 - a^6 + 4a^4(b^2 + c^2) \\ - 5a^2(b^4 + c^4) - 5a^2(b^4 + c^4) + b^2c^2(b^2 + c^2 - 3a^2) \\ \pm (a^2 - b^2)(c^2 - a^2) \sqrt{[3(2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4)]}.$$

If k = the area of a triangle whose sides are a , b , and c , then $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4 = 16k^2$.

$$\text{Hence, } 6(a^4 + b^4 + c^4 - b^2c^2 - c^2a^2 - a^2b^2)x^2 \\ = 2b^6 + 2c^6 - a^6 + 4a^4(b^2 + c^2) - 5a^2(b^4 + c^4) \\ b^2c^2(b^2 + c^2 - 3a^2) \pm 4(a^2 - b^2)(c^2 - a^2)k \sqrt{3}.$$

Hence, by permuting the letters a , b , c we can find the values of y^2 and z^2 from the two following equations:

$$6(a^4 + b^4 + c^4 - b^2c^2 - c^2a^2 - a^2b^2)y^2 = 2c^6 + 2c^6 + 2a^6 - b^6 + 4b^4(c^2 + a^2) \\ - 5b^2(c^4 + a^4) + c^2a^2(c^2 + a^2 - 3b^2) \pm 4(b^2 - c^2)(a^2 - b^2)(k \sqrt{3}) \\ \text{and } 6(a^4 + b^4 + c^4 - b^2c^2 - c^2a^2 - a^2b^2)z^2 = 2a^6 + 2b^6 - c^6 + 4c^4(a^2 + b^2) \\ - 5c^2(a^4 + b^4) + a^2b^2(a^2 + b^2 - 3c^2) \pm 4(c^2 - a^2)(b^2 - c^2)k \sqrt{3}.$$

II. Solution by ARTEMAS MARTIN, LL. D., Editor and Publisher, Mathematical Magazine, Washington, D. C.

From the square of the sum of the given equations, subtract twice the sum of their squares and extract the square root of one-third of the remainder; then

$$xy + yz + xz = \pm \frac{1}{3} \sqrt{[3(a^2 + b^2 + c^2)^2 - 6(a^4 + b^4 + c^4)]} \dots (4).$$

Subtracting twice (1) from the sum of (4) added to the sum of the given equations,

$$2x(x + y + z) = b^2 + c^2 - a^2 \pm \frac{1}{3} \sqrt{[3(a^2 + b^2 + c^2)^2 - 6(a^4 + b^4 + c^4)]} \dots (5).$$

Subtracting twice (2) and twice (3) in succession from the same sum,

$$2z(x + y + z) = a^2 + c^2 - b^2 \pm \frac{1}{3} \sqrt{[3(a^2 + b^2 + c^2)^2 - 6(a^4 + b^4 + c^4)]} \dots (6),$$

$$2z(x + y + c) = a^2 + b^2 - c^2 \pm \frac{1}{3} \sqrt{[3(a^2 + b^2 + c^2)^2 - 6(a^4 + b^4 + c^4)]} \dots (7).$$

Add the three equations (5), (6), and (7); then

$$2(x + y + z)(x + y + z) = 2(x + y + z)^2 = a^2 + b^2 + c^2 \\ \pm \sqrt{[3(a^2 + b^2 + c^2)^2 - 6(a^4 + b^4 + c^4)]} \dots (8).$$

Extracting the square root of twice (8),

$$2(x+y+z) = \pm \sqrt{\{2(a^2+b^2+c^2) \pm 2\sqrt{[3(a^2+b^2+c^2)^2 - 6(a^4+b^4+c^4)]}\}} \dots (9).$$

Dividing (5), (6), and (7) in succession by (9),

$$x = \frac{b^2+c^2-a^2 \pm \frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2 - (a^4+b^4+c^4)]}}{\pm \sqrt{\{2(a^2+b^2+c^2) \pm 2\sqrt{[3(a^2+b^2+c^2)^2 - (a^4+b^4+c^4)]}\}}},$$

$$y = \frac{a^2+c^2-b^2 \pm \frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2 - (a^4+b^4+c^4)]}}{\pm \sqrt{\{2(a^2+b^2+c^2) \pm 2\sqrt{[3(a^2+b^2+c^2)^2 - (a^4+b^4+c^4)]}\}}},$$

$$z = \frac{a^2+b^2-c^2 \pm \frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2 - (a^4+b^4+c^4)]}}{\pm \sqrt{\{2(a^2+b^2+c^2) \pm 2\sqrt{[3(a^2+b^2+c^2)^2 - (a^4+b^4+c^4)]}\}}}.$$

Also solved by A. H. Holmes, J. Scheffer, and V. M. Spunar.

For a number of different solutions of this problem, when the known quantities are not squared, see *The Mathematical Magazine*, published by Dr. Artemas Martin, Vol. II, pp. 141-144, and pp. 193-196. Ed. F.

GEOMETRY.

375. Proposed by C. N. SCHMALL, New York City.

From a point P on a circle there are drawn three chords PA , PB , PC . Show that the circles described on these chords as diameters intersect again in three collinear points.

I. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology.

Take the point P as the origin of polar coördinates, and the diameter through P as the initial line. The coördinates of the points A , B , and C are, respectively, $2a \cos \alpha$, α ; $2a \cos \beta$, β ; $2a \cos \gamma$, γ ; α , β , and γ being the vectorial angles.

The equations of the circles described upon the chords as diameters will be

$$\rho = 2a \cos \alpha \cos(\theta - \alpha),$$

$$\rho = 2a \cos \beta \cos(\theta - \beta),$$

$$\rho = 2a \cos \gamma \cos(\theta - \gamma),$$

whence the coördinates of the points of intersection are

$$2a \cos \beta \cos \gamma, \beta + \gamma; \quad 2a \cos \gamma \cos \alpha, \gamma + \alpha; \quad 2a \cos \alpha \cos \beta, \alpha + \beta.$$

These points are all on the straight line whose equation is